Online Appendix for "Risk and Information in Dispute Resolution: An Empirical Study of Arbitration"

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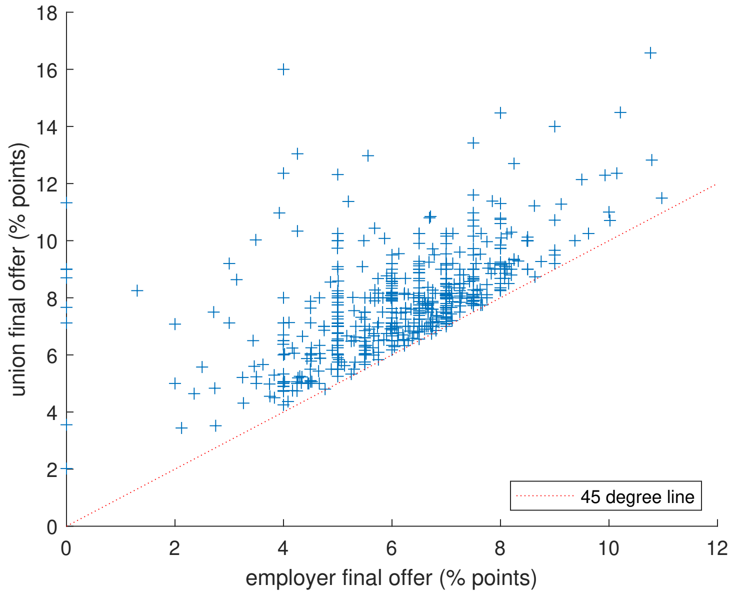
Appendix A. Supplementary Tables and Figures

Table A1. Offer Aggressiveness and Employer Win Probability, 1978-1995

	(1)	(2)	(3)
Union final offer residual	0.218		0.140
	(0.043)		(0.049)
Employer final offer residual		0.242	0.169
		(0.046)	(0.052)
Constant	-0.324	-0.334	-0.333
	(0.054)	(0.054)	(0.054)
Observations	579	579	579

Notes: Table reports Probit results. The unit of observation is a case. In all specifications, the sample consists of cases from the ARB_F data set, which are resolved by final-offer arbitration. The dependent variable is a dummy indicating whether the employer wins the arbitration. The regressors are residuals of regressions of the final offers by the union and the employer on all the covariates in column (1) of Table 2. Standard errors provided in parentheses.

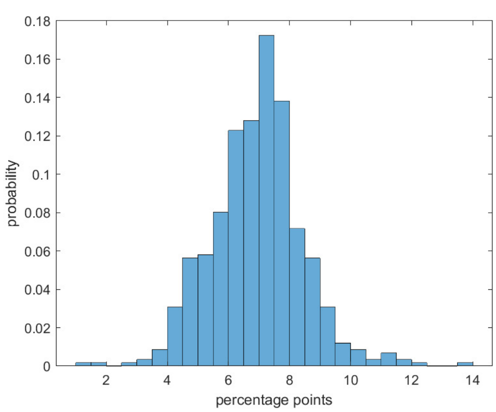
Figure A1. Scatter Plot of Final Offers, 1978–1995



Notes: Employer and union final offers in all cases from the ARB_F data set. The 45 degree line is marked with a dotted line.

Figure A2. Histograms of Arbitration Data

(a) Midpoint of Union and Employer FOA Offers, 1978–1995



(b) CA Arbitrated Wages, 1996–2000

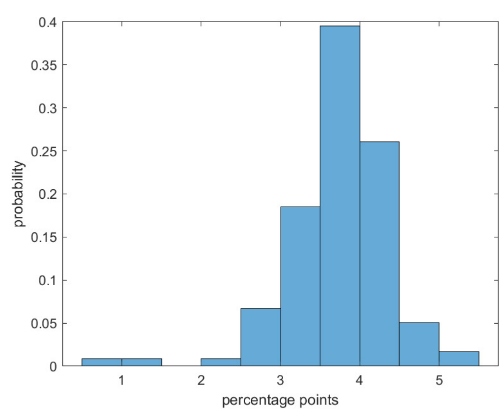
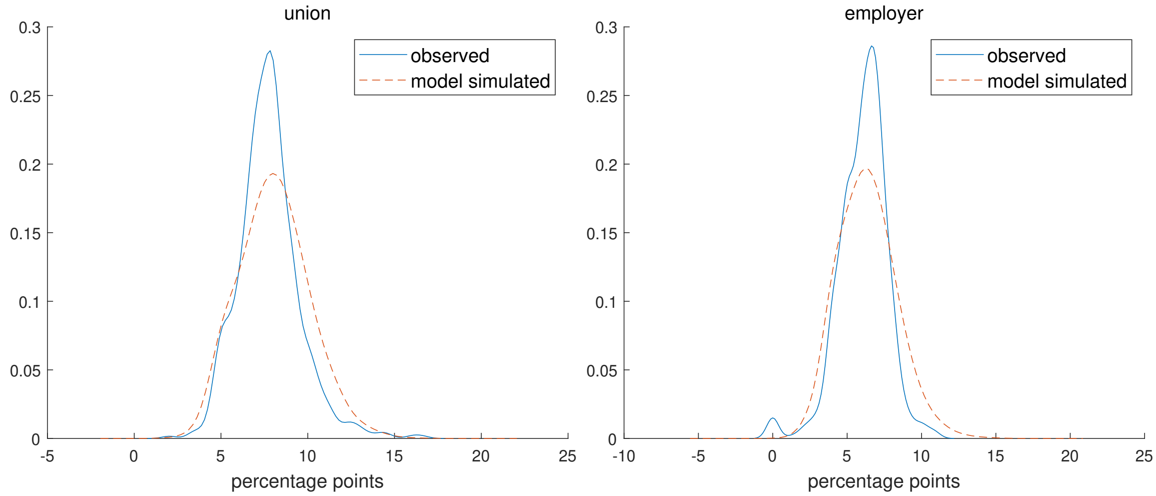
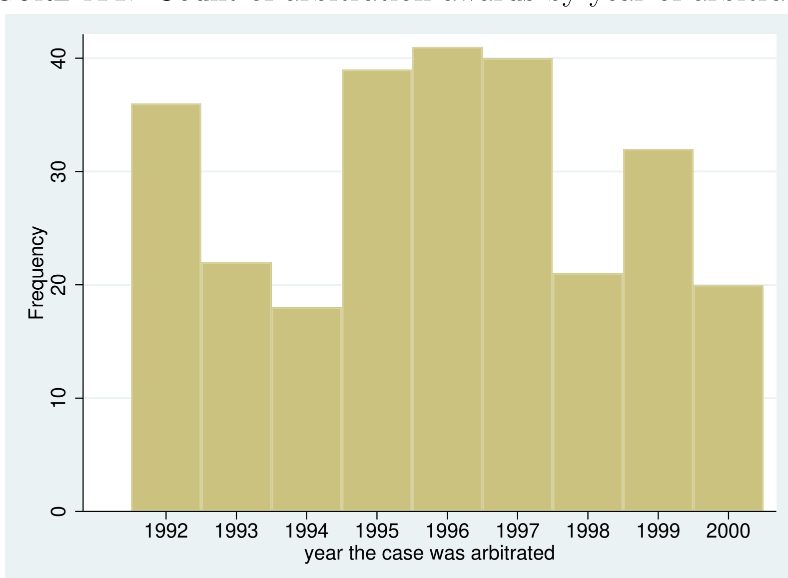


Figure A3. Model Fit: Final Offers, 1978-1995



Notes: Figures display kernel density of observed vs. model-simulated final offers by the union and the employer, respectively.

Figure A4. Count of arbitration awards by year of arbitration



Notes: The policy change from FOA to CA occurred in 1996.

Appendix B. FOA and CA: Supplementary Evidence

As a complement to the counterfactual analysis presented in Section 6.1, this Appendix compares the arbitration outcomes in final-offer (FOA) and conventional arbitration (CA) using a descriptive regression exercise. Recall that, in our setting, FOA was the default dispute resolution method until 1995, whereas, from 1996 onward, cases were resolved by CA. We exploit this institutional change in the following specification:

$$Outcome_i = \mu_0 + \mu_1 Conventional_i + \mu_2 X_i + \iota_i, \tag{A.1}$$

where the unit of observation is a case, denoted by i, and ι_i is an error term. As the dependent variable, $Outcome_i$, we consider the analogs of Table 4 outcomes, namely: (i) the difference between the offers made by the union and the employer; (ii) the difference between the wage increase decided by the arbitrator and the midpoint of the offers made by the parties; and (iii) the arbitrated wage increase. The regressor of interest is $Conventional_i$, a dummy that indicates whether case i is decided after 1996—that is, by CA. The vector X_i contains all of the covariates included in column (1) of Table 2 in the main text, except for the year-group fixed effects. Instead of controlling for year groups, we estimate (A.1) using only data on cases resolved from 1993 onward, so the FOA data used in the regression analysis constitutes only the last year group from the estimation sample employed in the main text (see Section 2.3 for information on the year-group fixed effects).

Table A2 presents OLS estimates of (A.1). Relative to FOA, CA is associated with a wider gap between the offers made by the union and the employer, as shown in column (1). Column (2) shows that, taking the midpoint between the parties' offers as a reference, the awards chosen by the arbitrator are smaller in CA than in FOA.[1]

[1] In FOA, the award tends to be above the offer midpoint because the union wins arbitration more often than the employer. In CA, the concept of one party winning or losing does not apply. But we can assess whether, in expectation, the award is closer to the union's offer or to that by the

Column (3) shows that CA cases are associated with a lower absolute arbitrated wage increase than are FOA cases. These findings mirror our results from Section 6.1, albeit with Section 6.1 estimating an effect of smaller magnitude for column (3).

It is worth stressing that, besides the obvious methodological distinctions, the regression presented in this Appendix and the counterfactual analysis in Section 6.1 are based on different samples. The latter provides a comparison between *observed* CA cases post-1996 and FOA outcomes that are *simulated*, given the covariates of the *same* post-1996 cases. In contrast, the regressions presented here compare only observed cases—using 1993-1995 data on FOA cases and 1996-2000 data on CA cases, so that covariates of the cases would be different for the two arbitration designs. Thus, "differences" between results of the two analyses need not imply a contradiction. In Table A2, we think column (3) would be the most influenced by these differences in covariate samples, while columns (1) and (2) would be more robust because they examine outcomes that constitute within-case differences. Indeed, Section 6.1, by enabling a comparison of arbitration designs given the *same* set of covariates, informs us that the estimated OLS coefficient in Table A2, column (3) may be exaggerated in magnitude.

Overall, the results from the reduced-form and structural approaches corroborate each other here and provide further credibility to the subsequent analyses in the main text that are motivated by these comparisons.

employer, and by how much. This is the purpose of comparing the award to the midpoint of offers, as we do in Table A2, column (2).

Table A2. FOA vs. CA: Offers and Case Outcomes (1993-2000)

	(1)	(2)	(3)
	Difference between Offers	Arb. Wage - Offer Midpoint	Arbitrated Wage Increase
Conventional	1.832	-0.357	-0.802
	(0.319)	(0.188)	(0.163)
Observations	158	158	158
R^2	0.394	0.175	0.416
Adjusted R^2	0.280	0.019	0.305

Controls: number of years covered by the contract; 12-month percent change in the CPI; *othermuni* (see Section 2.3 in main text for details); log of taxable property per capita; quantile rank of median household income among NJ municipalities; log of population; population density; a dummy indicating a contract for fire officers; a dummy indicating that the employer is a county; and the credit rating assigned to municipal debt obligations by Moody's Investors' Service.

Notes: Table reports OLS results. The unit of observation is a case. In all specifications, the sample consists of cases decided by final-offer arbitration (ARB_F data) from 1993-1995 and cases resolved by conventional arbitration (ARB_C data) from 1996-2000. The regressor of interest is a dummy indicating whether the case was decided by conventional arbitration. Standard errors provided in parentheses.

Appendix C. Proofs

Proof of Proposition 1. We adopt a "guess and verify" approach for the proof. Assume that offers take the form $y_u(s_p) = M_p(s_p) + \delta_u$ and $y_e(s_p) = M_p(s_p) - \delta_e$, where δ_u and δ_e do not depend on s_p.

First, we characterize the arbitrator's inference and the decision rule that best responds to the supposed $y_u(s_p)$, $y_e(s_p)$. As derived in the text following Proposition 1, the arbitrator's best response given the supposed $y_u(s_p)$, $y_e(s_p)$ is to infer s_p by the inference rule

$$s_p(\bar{y}) = \frac{(h + h_\varepsilon)\,[\bar{y} + (\delta_e - \delta_u)/2] - hm}{h_\varepsilon}.$$

Also, as derived in the text, the arbitrator then chooses y_e if and only if

$$s_a < \frac{h_\varepsilon \bar{y} + h(\bar{y} - m) + h_\varepsilon\,(\bar{y} - s_p(\bar{y}))}{h_\varepsilon} = \bar{y} - \left(\frac{h + h_\varepsilon}{h_\varepsilon}\right)\frac{\delta_e - \delta_u}{2} \equiv S(\bar{y}).$$

Second, we confirm that there exists a unique pair δ_u, δ_e such that the final offer strategies $y_u(s_p) = M_p(s_p) + \delta_u$ and $y_e(s_p) = M_p(s_p) - \delta_e$ in turn best respond to the inference and decision rules above and to one another. By Assumption 1, the parties' belief about the distribution of s_a conditional on s_p is normal with mean $M_p(s_p)$ and precision $H = [h_\varepsilon(h + h_\varepsilon)]\,/\,(h + 2h_\varepsilon)$. Let $\Phi(\cdot)$ and $\phi(\cdot)$ be the standard normal cumulative distribution and density functions, respectively. Then the decision rule above implies that the arbitrator selects y_e with probability $\Phi([S(\bar{y}) - M_p(s_p)]\sqrt{H})$.

We can then rewrite the problems solved by the union and the employer, respectively, as

$$\max_{\delta_u} u_u \left(M_p(s_p) - \delta_e\right)\Phi([S(\bar{y}) - M_p(s_p)]\sqrt{H}) + u_u \left(M_p(s_p) + \delta_u\right) \left[1 - \Phi([S(\bar{y}) - M_p(s_p)]\sqrt{H})\right],$$

$$\text{and } \max_{\delta_e} u_e \left(M_p(s_p) - \delta_e\right)\Phi([S(\bar{y}) - M_p(s_p)]\sqrt{H}) + u_e \left(M_p(s_p) + \delta_u\right) \left[1 - \Phi([S(\bar{y}) - M_p(s_p)]\sqrt{H})\right].$$

The corresponding first-order conditions are

$$\frac{\sqrt{H}}{2} \frac{\phi([S(\bar{y}) - M_p(s_p)]\sqrt{H})}{1 - \Phi([S(\bar{y}) - M_p(s_p)]\sqrt{H})} = \frac{\rho}{\exp\left(\rho(\delta_u + \delta_e)\right) - 1},$$

$$\text{and} \qquad \frac{\sqrt{H}}{2} \frac{\phi([S(\bar{y}) - M_p(s_p)]\sqrt{H})}{\Phi([S(\bar{y}) - M_p(s_p)]\sqrt{H})} = \frac{1}{\delta_u + \delta_e},$$

where we use the fact that the derivative of $S(\bar{y})$ with respect to the union's choice of δ_u and the employer's choice of δ_e are $1/2$ and $-1/2$, respectively.

In equilibrium, δ_u and δ_e must satisfy these FOCs with $M_p(s_p) = (\bar{y} + (\delta_e - \delta_u)/2)$. Plugging in this expression and rearranging, we find that the equilibrium δ_u and δ_e must satisfy

$$\frac{\sqrt{H}}{2} \frac{\phi\left(\eta(\delta_u - \delta_e)/2\right)}{1 - \Phi\left(\eta(\delta_u - \delta_e)/2\right)} = \frac{\rho}{\exp\left(\rho(\delta_u + \delta_e)\right) - 1},$$

$$\text{and} \qquad \frac{\sqrt{H}}{2} \frac{\phi\left(\eta(\delta_u - \delta_e)/2\right)}{\Phi\left(\eta(\delta_u - \delta_e)/2\right)} = \frac{1}{\delta_u + \delta_e},$$

where $\eta \equiv \sqrt{H}(h + 2h_\varepsilon)/h_\varepsilon$. These correspond to (4) and (5) in the text.

To show that there exists a unique pair δ_u, δ_e that solves the system of equations implied by these first-order conditions, define shorthand $t \equiv \eta(\delta_u - \delta_e)/2$, $d_1 \equiv \delta_u + \delta_e$,

$f(d_1) \equiv \rho/(\exp(\rho d_1) - 1)$, $\lambda \equiv \phi/(1 - \Phi)$ and $\tilde{\lambda} \equiv \phi/\Phi$. We can rewrite (4) and (5) as

$$\frac{\sqrt{H}}{2}\lambda(t) = f(d_1) \quad \text{and} \quad \frac{\sqrt{H}}{2}\tilde{\lambda}(t) = 1/d_1. \tag{A.2}$$

This system admits a solution in $t \in \mathbb{R}$ and $d_1 \in \mathbb{R}_+$ if and only if

$$\frac{\sqrt{H}}{2}\lambda(t) = f\left(\frac{2}{\sqrt{H}\tilde{\lambda}(t)}\right) \tag{A.3}$$

admits a solution in $t \in \mathbb{R}$. By construct, λ is increasing, while $\tilde{\lambda}$ and f are decreasing in t and d_1, respectively. As $t \to -\infty$, we know that $\lambda(t) \to 0$, $\tilde{\lambda}(t) \to \infty$, and the r.h.s of (A.3) diverges to ∞. On the other hand, as $t \to \infty$, we have that $\lambda(t) \to \infty$, $\tilde{\lambda}(t) \to 0$, and the r.h.s. of (A.3) converges to 0. Therefore both sides of (A.3) are strictly monotonic in different directions, implying existence of a unique solution in t. Given t, (A.2) pins down a unique d_1. Then, since t determines the difference between δ_u and δ_e and d_1 determines their sum, existence and uniqueness of t and d_1 yields existence and uniqueness of the values of δ_u and δ_e that satisfy (4) and (5).

Finally, as s_p is absent from (4) and (5), we verify that neither δ_u nor δ_e vary with the parties' signal s_p. □

Proof of Proposition 2. (i) Let $d_1 \equiv \delta_u + \delta_e$, the distance between final offers. In a proof by contradiction, suppose $h' > h$ and $d_1(h') \geq d_1(h)$. As the right-hand sides of (A.2) both decrease in d_1, we have $\sqrt{H(h')}\lambda(t(h')) \leq \sqrt{H(h)}\lambda(t(h))$ and $\sqrt{H(h')}\tilde{\lambda}(t(h')) \leq \sqrt{H(h)}\tilde{\lambda}(t(h))$. Since H is strictly increasing in h, this is only possible if $\lambda(t(h')) < \lambda(t(h))$ and $\tilde{\lambda}(t(h')) < \tilde{\lambda}(t(h))$. However, by definition, $\lambda(\cdot)$ is strictly increasing, while $\tilde{\lambda}(\cdot)$ is strictly decreasing, so it is impossible for these two inequalities to be satisfied simultaneously. Therefore, $d_1(h') < d_1(h)$ by contradiction. Repeat the same proof replacing h with h_ε to show that d_1 is strictly decreasing in h_ε.

(ii) While we use risk-neutrality for the employer and CARA utility for the union throughout this paper, here we relax the employer's risk-neutrality to prove a more general point. Let $U_u(\cdot)$ and $U_e(\cdot)$ be notation for the parties' CARA utility functions, which may differ in their risk aversion parameters. Taking a ratio of (4) and (5) yields

$$\frac{\Phi\left(\eta(\delta_u - \delta_e)/2\right)}{1 - \Phi\left(\eta(\delta_u - \delta_e)/2\right)} = \left(\frac{U_e(-y_e) - U_e(-y_u)}{U_u(y_u) - U_u(y_e)}\right) \frac{U_u'(y_u)}{U_e'(-y_e)}. \tag{A.4}$$

Now define a function $\tilde{U}_e(\cdot)$ such that $\tilde{U}_e(z + (y_u + y_e)) \equiv U_e(z)$. Note that, in terms of absolute risk aversion, if $U_u(\cdot)$ is more (less) risk-averse than $U_e(\cdot)$, it is also more (less) risk-averse than $\tilde{U}_e(\cdot)$. We can rewrite the equation above as

$$\frac{\Phi\left(\eta(\delta_u - \delta_e)/2\right)}{1 - \Phi\left(\eta(\delta_u - \delta_e)/2\right)} = \left(\frac{\tilde{U}_e(y_u) - \tilde{U}_e(y_e)}{U_u(y_u) - U_u(y_e)}\right) \frac{U_u'(y_u)}{\tilde{U}_e'(y_u)}.$$

By equation (22) in Pratt (1964), the r.h.s. of the above equation is < 1 if the union is more risk-averse, $= 1$ if the parties are equally risk-averse, and > 1 if the employer is more risk-averse. Then by the l.h.s. of the equation and properties of the standard normal cdf $\Phi(\cdot)$, $\delta_u < \delta_e$ if the union is more risk-averse, $\delta_u = \delta_e$ if the parties are equally risk-averse, and $\delta_u > \delta_e$ if the employer is more risk-averse.

Meanwhile, the l.h.s. above is the odds of the employer winning, by definition. Thus, the more risk-averse party wins more often in expectation. This proof is closely related to that of Farber (1980). □

Proof of Proposition 3. Denote the final offers by the union and the employer, respectively, by $y_u(s_p, h_\varepsilon)$ and $y_e(s_p, h_\varepsilon)$. From Proposition 1, we have $y_u\left(s_p, h_\varepsilon\right) = M_p\left(s_p, h_\varepsilon\right) + \delta_u\left(h_\varepsilon\right)$ and $y_e\left(s_p, h_\varepsilon\right) = M_p\left(s_p, h_\varepsilon\right) - \delta_e\left(h_\varepsilon\right)$. Define $d_1(h_\varepsilon) \equiv y_u\left(s_p, h_\varepsilon\right) - y_e\left(s_p, h_\varepsilon\right) = \delta_u\left(h_\varepsilon\right) + \delta_e\left(h_\varepsilon\right)$ and $d_2(h_\varepsilon) \equiv \left(\delta_u\left(h_\varepsilon\right) - \delta_e\left(h_\varepsilon\right)\right)$. Also, by (6), in equilibrium the arbitrator chooses the employer's final offer with probability $\Phi\left(\eta\left(h_\varepsilon\right)\left(\delta_u\left(h_\varepsilon\right) - \delta_e\left(h_\varepsilon\right)\right)/2\right)$, where $\eta\left(h_\varepsilon\right) \equiv \sqrt{H\left(h_\varepsilon\right)}\left(h + 2h_\varepsilon\right)/h_\varepsilon$ and $H\left(h_\varepsilon\right) \equiv h_\varepsilon\left(h + h_\varepsilon\right)/\left(h + 2h_\varepsilon\right)$.

First, we show that ρ is identified. From (7), we have

$$\frac{\Phi\left(\eta\left(h_{\varepsilon}\right) d_{2}\left(h_{\varepsilon}\right) / 2\right)}{1-\Phi\left(\eta\left(h_{\varepsilon}\right) d_{2}\left(h_{\varepsilon}\right) / 2\right)}=\frac{\rho d_{1}\left(h_{\varepsilon}\right)}{\exp \left(\rho d_{1}\left(h_{\varepsilon}\right)\right)-1}.$$

Let $odds\left(y_{u}-y_{e}\right)$ denote the observed odds that the employer's final offer is chosen by the arbitrator, conditional on the observed offer difference $y_u - y_e$. Proposition 2(i) shows that $d_1(h_\varepsilon)$ is strictly decreasing in h_ε, allowing us to use $h_{\varepsilon}=d_{1}^{-1}\left(y_{u}-y_{e}\right)$ and write

$$odds\left(y_{u}-y_{e}\right)=\frac{\Phi\left(\eta\left(d_{1}^{-1}\left(y_{u}-y_{e}\right)\right) d_{2}\left(d_{1}^{-1}\left(y_{u}-y_{e}\right)\right) / 2\right)}{1-\Phi\left(\eta\left(d_{1}^{-1}\left(y_{u}-y_{e}\right)\right) d_{2}\left(d_{1}^{-1}\left(y_{u}-y_{e}\right)\right) / 2\right)}. \tag{A.5}$$

Together, the equations above imply

$$odds\left(y_{u}-y_{e}\right)=\frac{\rho\left(y_{u}-y_{e}\right)}{\exp \left(\rho\left(y_{u}-y_{e}\right)\right)-1}. \tag{A.6}$$

From Theorem 1 and equation (22) in Pratt (1964), the r.h.s. is strictly decreasing in ρ, so the equation above identifies this parameter.

Next, we show the identification of h and $G_{h_\varepsilon}(\cdot)$. First, since $\Phi(x)/\left[1-\Phi(x)\right]$ is strictly increasing in x, (A.5) identifies the product $\eta\left(d_{1}^{-1}\left(y_{u}-y_{e}\right)\right) d_{2}\left(d_{1}^{-1}\left(y_{u}-y_{e}\right)\right)$. Plugging this value into the left-hand side of (4) then identifies $H\left(d_{1}^{-1}\left(y_{u}-y_{e}\right)\right)$, as the r.h.s. of that equation is a ratio of two identified terms. Rearranging the definition of $H\left(h_{\varepsilon}\right)$ gives

$$\frac{1}{H\left(h_{\varepsilon}\right)}=\frac{1}{h_{\varepsilon}}+\frac{1}{h+h_{\varepsilon}}=\frac{h}{h_{\varepsilon}}\left(\frac{1}{h}+\frac{1}{h} \frac{1}{1+\frac{h}{h_{\varepsilon}}}\right). \tag{A.7}$$

Meanwhile, from the definition of $M_{p}\left(s_{p}, h_{\varepsilon}\right)$, we have that

$$\operatorname{Var}\left[M_{p}\left(s_{p}, h_{\varepsilon}\right) | h_{\varepsilon}\right]=\left(\frac{h_{\varepsilon}}{h+h_{\varepsilon}}\right)^{2} \operatorname{Var}\left[s_{p} | h_{\varepsilon}\right]=\frac{1}{h}\left(\frac{1}{1+\frac{h}{h_{\varepsilon}}}\right), \tag{A.8}$$

where the l.h.s. is an observed quantity because

$$\begin{aligned}\text{Var}\left[M_p\left(s_p,h_\varepsilon\right)|h_\varepsilon=d_1^{-1}(y_u-y_e)\right] &= \text{Var}\left[y_u\left(s_p,h_\varepsilon\right)-\delta_u\left(h_\varepsilon\right)|h_\varepsilon=d_1^{-1}(y_u-y_e)\right]\\ &= \text{Var}\left[y_u\left(s_p,h_\varepsilon\right)|h_\varepsilon=d_1^{-1}(y_u-y_e)\right]\\ &= \text{Var}\left[y_u|y_u-y_e\right].\end{aligned}$$

Equations (A.14) and (A.8) thus form a system of equations that can be solved for h and h_ε. Specifically, we rearrange (A.8) as

$$\frac{h}{h_\varepsilon}=\frac{1}{h\text{Var}\left[y_u|y_u-y_e\right]}-1.$$

Plugging this into (A.7) gives

$$\frac{1}{H\left(d_1^{-1}\left(y_u-y_e\right)\right)}=\left(\frac{1}{h\text{Var}\left[y_u|y_u-y_e\right]}-1\right)\left(\frac{1}{h}+\text{Var}\left[y_u|y_u-y_e\right]\right),$$

which corresponds to (8) in the text. The only unknown in the equation above is h, and the right-hand side is strictly decreasing in this parameter. Hence, this equation identifies h, which, in turn, identifies h_ε by (A.8). As the distribution of y_u-y_e is observed, and we identify $h_\varepsilon=d_1^{-1}\left(y_u-y_e\right)$ for any value of y_u-y_e, we have nonparametric identification of $G_{h_\varepsilon}(\cdot)$.

Identification of h and h_ε implies identification of $\eta\left(h_\varepsilon\right)$. Then $d_2\left(h_\varepsilon\right)$ is identified since the product $\eta\left(h_\varepsilon\right)d_2\left(h_\varepsilon\right)$ is known. So we know both $d_2\left(h_\varepsilon\right)$ and $d_1\left(h_\varepsilon\right)$, implying recovery of $\delta_u\left(h_\varepsilon\right)$ and $\delta_e\left(h_\varepsilon\right)$ for all h_ε in the support of $G_{h_\varepsilon}(\cdot)$.

Finally, we identify the parameter m. We have

$$\text{E}\left[M_p\left(s_p,h_\varepsilon\right)\right]=\text{E}\left[\text{E}\left[M_p\left(s_p,h_\varepsilon\right)|h_\varepsilon\right]\right]=\text{E}\left[\frac{hm+h_\varepsilon\text{E}\left[s_p|h_\varepsilon\right]}{h+h_\varepsilon}\right]=m.$$

Therefore, we have

$$\begin{aligned} m &= \mathrm{E}\left[\mathrm{E}\left[M_p\left(s_p, h_\varepsilon\right)|h_\varepsilon\right]\right] \\ &= \mathrm{E}\left[\mathrm{E}\left[y_u - \delta_u\left(h_\varepsilon\right)|h_\varepsilon\right]\right], \end{aligned}$$

where the right-hand side is now known. □

Identifying the Employer's Risk Attitude. Suppose we allow CARA utility for both the union and the employer, so that ρ_u and ρ_e are the union's and employer's CARA parameters, respectively. By equation (A.4), the odds of the employer winning case i in equilibrium equals

$$\frac{\exp(\rho_e d_{1i}) - 1}{\exp(\rho_u d_{1i}) - 1}\frac{\rho_u}{\rho_e},$$

where d_{1i} is the difference between union and employer final offers in case i. Given variation in d_{1i}, the expression above yields many identifying equations, allowing estimation of both ρ_u and ρ_e as long as $\rho_u \neq \rho_e$. Estimating ρ_u and ρ_e using a minimum distance estimator based on the above, we obtain $\hat{\rho}_e \approx 0$.

Appendix D. Asymmetric Signal Precision

In this section, we extend the model in Section 3 to allow the arbitrator's signal precision to be *different* from that of the parties. Formally, we relax part (iii) of Assumption 1 as follows:

Assumption A1. *Part (i) and (ii) in Assumption 1 hold; (iii) the distributions of* ε_a *and* ε_p *are normal with mean zero and precision* h_a *and* h_p*, respectively.*

All other model elements remain the same as in Section 3. We characterize the equilibrium of this new model with asymmetric signal precision, prove identification of the model primitives and estimate them. We collect relevant proofs in Section D.4, focusing on differences from the original proofs to avoid repetition of text.

D.1. **Equilibrium.** When it comes to the model equilibrium and its properties summarized by Propositions 1 and 2, we find that the same conclusions hold as before, with h_p and h_a appropriately replacing h_ε. Defining $\tilde{M}_p(s_p) \equiv (hm + h_p s_p)/(h + h_p)$, we restate the analogous propositions in terms of h_p and h_a below.

Proposition A1. *Under Assumption A1, there exists a separating Perfect Bayesian Equilibrium of the arbitration game in which the final offers by the union and the employer have the form* $y_u(s_p) = \tilde{M}_p(s_p) + \tilde{\delta}_u$ *and* $y_e(s_p) = \tilde{M}_p(s_p) - \tilde{\delta}_e$*. The terms* $\tilde{\delta}_u$ *and* $\tilde{\delta}_e$ *are unique and do not depend on the signal* s_p*.*

Proposition A2. *The equilibrium characterized in Proposition A1 is such that: (i) the distance between final offers* $(\tilde{\delta}_u + \tilde{\delta}_e)$ *is strictly decreasing in the precision parameters* h*,* h_p *and* h_a*; and (ii) the more risk-averse party chooses a final offer that is less distant from* $\tilde{M}_p(s_p)$*—i.e., a smaller* $\tilde{\delta}$*—and wins more often in expectation.*

D.2. **Identification and Estimation.**

D.2.1. *Identification.* Consider the data-generating process described in Section 4.1 with the following modifications. First, we treat the precision of the signals received by the parties, $h_{p,i}$, as a random variable drawn independently across cases i from a distribution $G_{h_p}(\cdot)$. Second, we let $h_{a,i} = \mu h_{p,i}$ for a constant $\mu > 0$ that represents how much more/less precise the arbitrator's signal is relative to that of the parties.

In this model, the primitives include: the union's risk aversion parameter, ρ; the parameters of the ideal wage increase distribution, m and h; the distribution of the parties' signal precision, $G_{h_p}(\cdot)$; and the arbitrator-to-parties precision ratio, μ.

Proposition A3. *Under Assumption A1 and the equilibrium of Proposition A1, the model primitives ρ, m, h, μ and the distribution $G_{h_p}(\cdot)$ are identified from the joint distribution of final offers (y_u, y_e) and the arbitrator's decision y.*

The intuition for identifying the model parameters other than the newly added μ is analogous to that presented in Section 4.2. As for μ, the arbitrator's signal precision ratio, it is identified separately from the parties' signal precision h_p because these have distinct effects on the final offer distribution. For simple intuition, consider the case of symmetric risk attitudes, given which the offer midpoint is exactly equal to $\tilde{M}_p(s_p)$. It is apparent that h_p affects both the variance of offer midpoints (because $\tilde{M}_p(s_p)$ is a function of h_p) and the distance between union and employer offers (by Proposition A2), while μ decreases the latter without affecting the former.

D.2.2. *Estimation Procedure.* Revisiting the key identifying equations from Section 5, equation (9) becomes

$$\tilde{H}_i \equiv \frac{\mu h_{p,i}\left[h_i + h_{p,i}\right]}{h_i + (1+\mu)h_{p,i}}. \tag{A.9}$$

Equation (11) now becomes

$$0 = \left[V_i\left(\frac{1}{\tilde{H}_i} + V_i\right)\right]^{\frac{1}{2}} - \left[\frac{1}{\mu h_i^2} + (1-\frac{1}{\mu})\frac{V_i}{h_i}\right]^{\frac{1}{2}}, \tag{A.10}$$

where V_i is shorthand for $\text{Var}\left(y_{u,i}|\tilde{d}_{1,i}, x_i\right)$ and $\tilde{d}_{1,i} \equiv y_{u,i} - y_{e,i}$. When $\mu = 1$, (A.10) simplifies to (11) because $\left[h_i^{-2} + (1-1)V_i/h_i\right]^{\frac{1}{2}} = 1/h_i$. Equation (12) now becomes

$$\tilde{d}_{2,i}(\mu) = \frac{\mu h_{p,i}\Phi^{-1}(p_i)}{\sqrt{\tilde{H}_i}\,[h_i + (1+\mu)h_{p,i}]} = \frac{\Phi^{-1}(p_i)}{\sqrt{\tilde{H}_i}} / \left[1 + \frac{1}{\mu}\left(\frac{1}{h_i V_i}\right)\right], \tag{A.11}$$

where the second equality uses $h_i/h_{p,i} + 1 = 1/(h_i V_i)$ (see equation (A.15)).

We estimate ρ using the same steps as in Section 5. For estimating m and h, we maintain the specifications $m(x_i;\theta_m) = x_i\theta_m$ and $h(x_i;\theta_h) = 1/\exp(x_i\theta_h)$. Then let $\zeta_{1,i}(\theta_h;\mu)$ refer to the right-hand side of (A.10) evaluated at $h_i = h(x_i;\theta_h)$. Now, *for any given value of* μ, we can estimate the remaining model parameters as follows. Based on (A.10), we first estimate $\theta_h(\mu)$ as

$$\hat{\theta}_h(\mu) \equiv \arg\min_{\theta_h} \sum_i \zeta_{1,i}(\theta_h;\mu)^2.$$

Let $\hat{\zeta}_{1,i}(\mu) \equiv \zeta_{1,i}(\hat{\theta}_h(\mu);\mu)$. Second, we estimate $h_{p,i}(\mu)$ for each arbitration case in the sample by solving for $h_{p,i}$ in (A.9). Third, defining

$$\zeta_{2,i}(\theta_m;\mu) \equiv \frac{y_{u,i} + y_{e,i}}{2} - \hat{d}_{2,i}(\mu) - m(x_i;\theta_m),$$

we estimate $\theta_m(\mu)$ as

$$\hat{\theta}_m(\mu) \equiv \arg\min_{\theta_m} \sum\nolimits_i \zeta_{2,i}(\theta_m;\mu)^2.$$

Let $\hat{\zeta}_{2,i}(\mu) \equiv \zeta_{2,i}(\hat{\theta}_m(\mu);\mu)$. Finally, we estimate μ by minimizing the criterion

$$\hat{\mu} \equiv \arg\min_{\mu} \sum\nolimits_i w_1\left[\hat{\zeta}_{1,i}(\mu)\right]^2 + w_2\left[\hat{\zeta}_{2,i}(\mu)\right]^2.$$

The weights w_1 and w_2 are the inverse of the empirical variance of $\zeta_{1,i}(\hat{\theta}_{h,0};\mu_0)$ and $\zeta_{2,i}(\hat{\theta}_{m,0};\mu_0)$, respectively, where μ_0 refers to the value of μ originally used in the main text, i.e., $\mu_0 = 1$, and $\hat{\theta}_{h,0}$, $\hat{\theta}_{m,0}$ refer to the original estimates reported in the main text given $\mu_0 = 1$. These serve the role of "first-step estimates" allowing us to obtain the weights w_1 and w_2.

D.3. **Estimated signal precision ratio μ.** Using the estimation procedure described in Section D.2.2, we obtain $\hat{\mu} = 0.90$ as the estimated ratio of arbitrator's signal precision $h_{a,i}$ to parties' signal precision $h_{p,i}$. The 95% bootstrap confidence interval of $\hat{\mu}$ based on 200 bootstrap samples is [0.70, 2.30].

D.4. **Proofs for Propositions in Appendix D.1 and D.2.**

Proof of Proposition A1. We adopt a "guess-and-verify" approach as in the proof of Proposition 1, assuming the parties' final-offer strategies take the form $y_u(s_p) = \tilde{M}_p(s_p) + \tilde{\delta}_u$ and $y_e(s_p) = \tilde{M}_p(s_p) - \tilde{\delta}_e$, where $\tilde{\delta}_u$ and $\tilde{\delta}_e$ do not depend on s_p. The proof follows the exact steps of the proof of Proposition 1 with the following new definitions and notation. The arbitrator's inference rule in equation (2) now becomes

$$\tilde{s}_p(\bar{y}) = \frac{(h + h_p)\left[\bar{y} + (\tilde{\delta}_e - \tilde{\delta}_u)/2\right] - hm}{h_p}. \tag{A.12}$$

The arbitrator's updated updated expectation of the ideal wage increase given s_a and $\tilde{s}_p(\bar{y})$ becomes

$$\tilde{y}_a(s_a, y_u, y_e) = \frac{hm + h_p\tilde{s}_p(\bar{y}) + h_a s_a}{h + h_p + h_a}.$$

The arbitrator's decision rule for selecting y_e (equation (3)) now becomes

$$s_a < \frac{h_a\bar{y} + h(\bar{y} - m) + h_p\left(\bar{y} - \tilde{s}_p(\bar{y})\right)}{h_a} = \bar{y} - \left(\frac{h + h_p}{h_a}\right)\frac{\tilde{\delta}_e - \tilde{\delta}_u}{2} \equiv \tilde{S}(\bar{y}). \tag{A.13}$$

Then the probability that the arbitrator selects y_e is $\Phi([\tilde{S}(\bar{y}) - \tilde{M}_p(s_p)]\sqrt{\tilde{H}})$, with $\tilde{H} \equiv (h_a(h+h_p))/(h+h_p+h_a)$. The rest of the proof follows the same argument as the proof of Proposition 1, with $M_p(s_p), \delta_e, \delta_u, H$ replaced by $\tilde{M}_p(s_p), \tilde{\delta}_e, \tilde{\delta}_u, \tilde{H}$ respectively, and with $\tilde{\eta} \equiv \sqrt{\tilde{H}}(h + h_p + h_a)/h_a$. □

Proof of Proposition A2. Proof of (i) is almost identical to that in Proposition 2. The only difference is that the last step now uses the fact that $\tilde{H}$ is increasing in both h_a and h_p, as well as in h. Proof of (ii) is identical to that in Proposition 2. $\square$

Proof of Proposition A3. Denote the final offers by the union and the employer by $y_u(s_p, h_p)$ and $y_e(s_p, h_p)$ respectively. Note that we now write h_p as an explicit argument in the final-offer strategies.

The first step is identify ρ, the risk aversion parameter in the union's utility. The argument for recovering ρ is almost identical to that in Proposition 3, only with $\delta_u(h_\varepsilon), \delta_e(h_\varepsilon), d_1(h_\varepsilon), d_2(h_\varepsilon)$ now replaced by $\tilde{\delta}_u(h_p), \tilde{\delta}_e(h_p), d_1(h_p) \equiv y_u(s_p, h_p) - y_e(s_p, h_p) = \tilde{\delta}_u(h_p) + \tilde{\delta}_e(h_p), d_2(h_p) \equiv \tilde{\delta}_u(h_p) - \tilde{\delta}_e(h_p)$ respectively, and with $\eta(h_\varepsilon)$ and $H(h_\varepsilon)$ now replaced by their counterparts:

$$\tilde{\eta}(h_p; \mu) \equiv \sqrt{\tilde{H}(h_p)}\,(h + h_p + \mu h_p) / (\mu h_p)$$

and

$$\tilde{H}(h_p; \mu) \equiv \mu h_p (h + h_p) / (h + h_p + \mu h_p).$$

The same argument as in Proposition 3 shows ρ is identified from (A.6), which by the proof of Proposition A1 also holds in this extended model with asymmetric signal precision.

The second step is to identify h and μ. Following the same argument as in the proof of Proposition 3, $\tilde{H}(h_p)$ is identified at $h_p = d_1^{-1}(y_u - y_e)$, and thus the following equation has a known left-hand side.

$$\frac{1}{\tilde{H}(h_p)} = \frac{1}{\mu h_p} + \frac{1}{h + h_p} = \frac{h}{h_p}\left(\frac{1}{\mu h} + \frac{1}{h}\frac{1}{1 + \frac{h}{h_p}}\right). \tag{A.14}$$

Meanwhile, from the definition of $\tilde{M}_p(s_p, h_p)$, we get

$$\text{Var}\left[\tilde{M}_p(s_p, h_p)\,|h_p\right] = \left(\frac{h_p}{h+h_p}\right)^2 \text{Var}\left[s_p|h_p\right] = \frac{1}{h}\left(\frac{1}{1+\frac{h}{h_p}}\right), \tag{A.15}$$

where the left-hand side is identified at $h_p = d_1^{-1}(y_u - y_e)$ as

$$\text{Var}\left[\tilde{M}_p(s_p, h_p)\,|h_p = d_1^{-1}(y_u - y_e)\right] = \text{Var}\left[y_u|y_u - y_e\right] \equiv v(\Delta y),$$

with $\Delta y \equiv y_u - y_e$. Rearranging (A.15) as

$$\frac{h}{h_p} = \frac{1}{hv(\Delta y)} - 1$$

and plugging it into (A.14) gives

$$I(\Delta y) = \left(\frac{a}{v(\Delta y)} - 1\right)\left[b + v(\Delta y)\right], \tag{A.16}$$

where $I(\Delta y) \equiv 1/\tilde{H}(d_1^{-1}(y_u - y_e))$, and $a \equiv 1/h, b \equiv 1/(\mu h)$ are fixed constants. Thus, in this extended model, the equality above must hold as Δy varies continuously over its equilibrium support.

We prove identification of (μ, h) by contradiction. Suppose (μ', h') is observationally equivalent to (μ, h). Then (A.16) must also hold for (μ', h'). This implies:

$$\frac{a - v(\Delta y)}{a' - v(\Delta y)} = \frac{b' + v(\Delta y)}{b + v(\Delta y)}$$

for all Δy over the support of offer differences in equilibrium. Differentiating both sides above w.r.t. $v(\Delta y)$ and equating the derivatives imply that $(a - a')$ and $(b - b')$ must have the same sign.[2] But this contradicts the supposition that (A.16) must hold for both (μ, h) and (μ', h') for all Δy. (To see this, note by construction $a/v(\Delta y) - 1$ and $b + v(\Delta y)$ are positive for all (a, b) or (a', b').) Hence, h and μ are identified.

[2]This argument requires the actual $v(\Delta y)$ in the data-generating process to vary continuously over at least some sections of its equilibrium support. But this follows immediately from continuous variation in h_p in our model.

The next step is to recover the distribution $G_{h_p}(\cdot)$. For each case in the sample, h_p can be recovered from the offer difference Δy using (A.15), where the left-hand side is a directly identifiable conditional variance $v(\Delta y)$. Thus, $G_{h_p}(\cdot)$ is identified nonparametrically.

The final step is to identify m. This follows from the same argument as in the last step in the proof of Proposition 3, only with $M_p(s_p, h_\varepsilon)$ replaced by $\tilde{M}_p(s_p, h_p)$, and $\delta(h_\varepsilon)$ replaced by $\tilde{\delta}(h_p)$. $\square$

Appendix E. Subsample Analysis

In this section, we repeat the estimation and counterfactual analyses after restricting the estimation sample to arbitration cases where both the union and the employer were represented by expert agents. The number of arbitration cases in this subsample is 313. The union is still estimated to be risk-averse, with parameter 0.32. Counterfactual results from the subsample analysis are presented below.

Table A3. Conventional Versus Final-Offer Arbitration, 1996-2000

	Conventional, observed	Final-offer, simulated
(a) Mean difference between offers	2.48	0.87
(b) Mean arbitrated wage − offer midpoint	-0.26	0.04
(c) Probability of union win	n/a	0.54
(d) Mean arbitrated wage increase	3.70	3.72

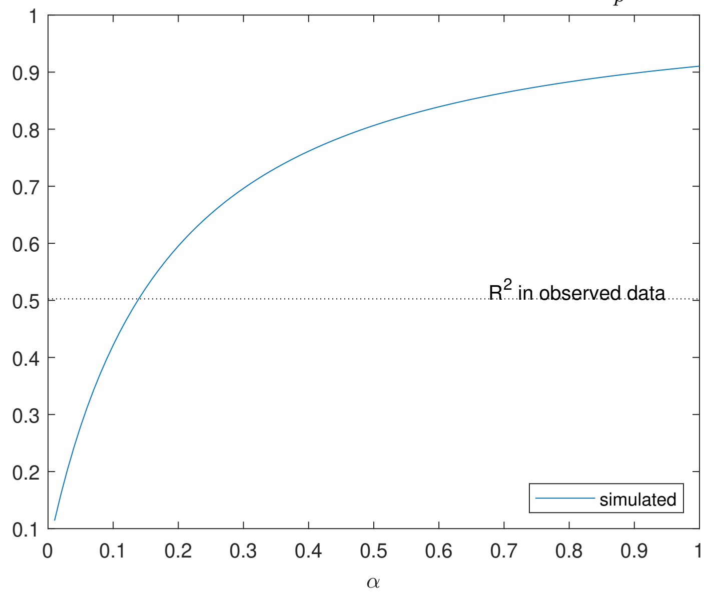
Notes: Column 1 shows average outcomes of the observations in ARB_C. Column 2 Monte Carlo simulates the arbitration model 1000 times conditional on each set of covariates in ARB_C. Offers and wage increases are in units of percentage points.

Table A4. Efficiency of Awards in CA and FOA

	Conventional $\alpha = 0.37$	Final-offer
$E[-(y-s)^2]$	-0.06	-0.16
$E[-\lvert y-s \rvert]$	-0.19	-0.32

Notes: The table displays the mean of the efficiency measure across 1000 Monte Carlo simulations conditional on each set of covariates in the ARB_C data set.

FIGURE A5. R^2 of Regressing y_a on s_p^*



Notes: Figure displays simulated R^2 values of regression (13) as a function of α, the degree of information transmission. At each value of α, we Monte Carlo simulate 1000 cases per each set of covariates observed in ARB_C and run the regression. For comparison, the dotted, horizontal line marks the R^2 of a regression analogous to (13) run using the observed data from ARB_C. The solid curve and dotted line intersect at $\alpha = 0.14$.

TABLE A5. Risk-Averse Versus Risk-Neutral Union in FOA, 1978–1995

	risk neutral	$\rho = 0.32$	$\rho = 1.5$
(a) Mean union offer	8.04	7.78	7.40
(b) Mean employer offer	6.16	6.28	6.34
(c) Probability of union win	0.50	0.56	0.70
(d) Mean arbitrated wage increase	7.10	7.20	7.15
(e) Union's certainty equivalent	7.10	6.70	5.54

Notes: The FOA model is Monte Carlo simulated 1000 times conditional on each set of covariates in the subset of the ARB_F data set where both union and employer were represented by an expert agent. Units are percentage points, excluding probabilities. Employer is risk neutral throughout.

Appendix F. Settlement and Selection into Arbitration

Not all collective negotiations of police and fire officer unions in New Jersey are arbitrated. Given that the main data set that we employ for the estimation of our structural model, ARB_F, consists exclusively of cases resolved through arbitration, an interesting question arises of whether selection affects our empirical results. Specifically, if the realization of the parties' signal about the ideal wage increase affects the odds that arbitration is required to resolve the dispute, the distribution of signals in our sample could differ from that in the general population of cases.

In this section, we explore this topic through additional theoretical and empirical analysis. We first specify a model of pre-arbitration negotiations that allows the parties to settle their dispute. By settling the case, the parties incur backing-down costs, which reflect a range of factors that affect the desirability of a settlement relative to taking the case to arbitration. Such backing-down costs are orthogonal to the pre-arbitration ideal-wage signals perceived by both parties. Our analysis here is quite general, in that we consider alternative versions of the bargaining model—with and without incomplete information between the parties regarding their mutual backing-down costs—which in turn lead to different bargaining solutions. In the case of complete information, our analysis actually accommodates a variety of solutions to the bargaining problem. Our interest is in characterizing the probability of settlement in each bargaining model. Despite the generality of the analysis, we are able to establish, in all versions of the model, a tight connection between the probability that the parties settle the case and the *difference* between their respective certainty equivalents of going to arbitration. Even if the levels of the certainty equivalents change, the settlement probability is not affected if their difference remains the same.

We then turn to our structural model of arbitration to empirically assess how the signal s_p affects the difference between the union's and employer's certainty equivalents of arbitration. Given the estimated model primitives reported in Section 5, we find that changes in the signal received by the parties have essentially no effect on their certainty-equivalent gap. According to our theoretical analysis, it then follows that the conditional probability of going to arbitration is invariant to changes in the signal s_p. Thus, the theoretical and empirical results in the present section show that the estimation results from the main text are fully consistent with a variety of data-generating processes that produce no systematic selection on signals. We proceed below with the theoretical results in Section F.1 and the empirical evidence in Section F.2.

F.1. **A Model of Pre-Arbitration Negotiations.** Prior to arbitration, the union and the employer have the opportunity to settle the case. Throughout the present section, we refer to this pre-arbitration interaction as the *negotiation stage*. In the absence of a settlement at the negotiation stage, the case proceeds to the arbitration stage, which consists of the model described in the main text. If the case reaches the arbitration stage, the dispute-resolution process therein eventually gives rise to a wage increase of y. At the negotiation stage, the parties are aware of their own signal, s_p, but they do not know s_a, the signal to be received by the arbitrator if the case proceeds to arbitration. Therefore, from the perspective of the union and the employer at the negotiation stage, y is a random variable.

The union and the employer incur *backing-down* costs c_u and c_e if they settle the dispute prior to arbitration. These costs may vary from one dispute to another. Specifically, for $j \in \{u, e\}$, c_j follows a distribution F_{c_j}. We assume that c_u and c_e are mutually independent from both y and s_p, and that each party knows the realization of its own backing-down costs at the beginning of the negotiation stage. The idea of backing-down costs in negotiations goes back to the classic contributions by Schelling (1956) and Crawford (1982). In our setting of disputes concerning salary increases, Reilly (1963) notes that taking a case all the way to arbitration can often be attractive to the negotiators because it allows them to give their clients the impression of having fought to the end while shifting responsibility to the arbitrator. The backing-down costs, as formulated here, can also include the positive aspects of a settlement, such as the monetary savings associated with the arbitrator and lawyer fees. We interpret backing-down costs flexibly as a term encompassing these various components that affect the desirability of a settlement relative to arbitration.

The negotiation stage payoff structure is as follows: Let $\tilde{y}_u \equiv \frac{-1}{\rho} \log \left(\mathrm{E} \left[\exp \left(-\rho y \right) \right] \right)$ be the union's certainty equivalent to obtaining the random wage increase y at the arbitration stage. Similarly, denote by $\tilde{y}_e \equiv \mathrm{E}[y]$ the expected arbitrated wage increase—that is, the negative of the (risk-neutral) employer's certainty equivalent of having y decided at arbitration. If negotiations break down and the parties proceed to arbitration, the expected payoffs, in certainty-equivalent terms, are $\tilde{y}_u$ for the union and $-\tilde{y}_e$ for the employer. If, instead, the parties agree to settle the dispute for a wage increase of σ, then the payoffs are $\sigma + c_u$ for the union and $-\sigma + c_e$ for the employer.

We next consider two alternative specifications of the negotiation stage game. In one of them, which we refer to as NEG_1, the realized backing-down costs, c_u and c_e, are common knowledge between the parties. This complete-information environment allows us to be agnostic about the bargaining protocol employed in the pre-arbitration negotiations; we only assume that the solution to the bargaining problem faced by the parties is efficient—that is, disputes only go into arbitration if the overall gains of settling them beforehand are negative. In a second specification, which we refer to as NEG_2, we consider the case of incomplete information between the parties regarding their backing-down costs. Having incomplete information gives rise to the possibility of inefficient bargaining breakdown, but it also requires us to impose a relatively simple bargaining protocol to keep the analysis tractable.

In both NEG_1 and NEG_2, we show that any change in the distribution of y that leaves the difference $\tilde{y}_u - \tilde{y}_e$ constant results in no change in the set of realized backing-down costs that are conducive to settlement. Together with the empirical findings presented in Section F.2, these results suggest that the structural estimates in the main text are not heavily affected by non-random selection on signals into our sample of arbitrated cases.

F.1.1. *NEG_1: Backing-Down Costs are Common Knowledge between the Parties.* If the backing-down costs are common knowledge between the union and the employer, it is natural to assume that the solution to their pre-arbitration negotiations is efficient. In other words, the parties settle the dispute if and only if their joint gains from settling are positive. Efficiency typically appears in cooperative bargaining solutions that are often adopted by the literature for the analysis of complete information bargaining games. In particular, the Nash bargaining solution (Nash Jr, 1950) has efficiency as one of its axioms. For the purposes of our analysis of the NEG_1 version of the negotiation stage, other than assuming efficiency, we maintain an agnostic view of the exact bargaining protocol adopted by the parties, as well as of the solution to their bargaining problem.

Given our assumption of efficiency, pre-arbitration negotiations break down when the parties' joint gains from settling are negative—that is, if the sum of the settlement payoffs for the union and the employer is smaller than the sum of the parties' certainty equivalents of going to arbitration. This condition boils down to

$$\sigma + c_u - \sigma + c_e < \tilde{y}_u - \tilde{y}_e,$$

which simplifies to

$$c_u + c_e < \tilde{y}_u - \tilde{y}_e.$$

Under the assumption of independence between the backing-down costs and y, the following proposition is self-evident:

PROPOSITION A4. *Given an initial equilibrium of NEG_1, the set of backing-down costs leading to arbitration is invariant to a change in the distribution of the potential arbitration awards that holds $\tilde{y}_u - \tilde{y}_e$ constant.*

A direct implication of Proposition A4 is that, upon any change in the distribution of potential arbitration awards that leaves $\tilde{y}_u - \tilde{y}_e$ constant, the equilibrium probability that the dispute reaches the arbitration stage also stays the same.

F.1.2. *NEG_2: Incomplete Information Regarding Backing-Down Costs.* We now consider the case in which the backing-down costs, c_u and c_e, are privately known by the parties. Specifically, only the union knows the realization of c_u, and only the employer knows the realization of c_e. We begin by making the following assumption about the distributions of c_u and c_e:

ASSUMPTION A2. *(i) For $j \in \{u, e\}$, F_{c_j} has an associated density function f_{c_j} such that $f_{c_j}(c) > 0$ over its entire support; and (ii) the hazard function associated with the union's cost distribution, $f_{c_u}(c)/[1 - F_{c_u}(c)]$, is strictly increasing in c.*

The bargaining protocol in NEG_2 is take-it-or-leave-it.[3] Specifically, the order of play in the negotiation stage is as follows: The union and employer draw their respective costs c_u and c_e. The employer then offers to settle the case for a wage increase σ. If the union rejects the offer, the case proceeds to the arbitration stage.

[3]Though stylized, the take-it-or-leave-it solution is relevant. Perry (1986) shows that in an alternating-offer game with two-sided incomplete information where the cost of bargaining takes the form of a fixed cost per period rather than discounting, the unique sequential equilibrium takes the form of a take-it-or-leave-it offer game.

We solve NEG_2 by backward induction. The union rejects a settlement offer σ if its certainty equivalent of going to arbitration, $\tilde{y}_u$, is greater than $\sigma + c_u$, its settlement payoff. This condition simplifies to

$$\sigma < \tilde{y}_u - c_u. \tag{A.17}$$

The employer does not know the union's c_u. Therefore, the employer's problem is

$$\max_{\sigma} F_{c_u}(\tilde{y}_u - \sigma)(-\tilde{y}_e) + [1 - F_{c_u}(\tilde{y}_u - \sigma)](-\sigma + c_e), \tag{A.18}$$

where $F_{c_u}(\tilde{y}_u - \sigma)$ is the probability that the union rejects settlement offer σ. We restrict our attention to interior solutions of the employer's problem—that is, offers that the union accepts with some probability in the interval $(0, 1)$. This restriction is for ease of exposition; but it also comprises the most relevant scenario, given our interest in the selection of cases into arbitration.

The first-order condition associated with (A.18) is

$$\sigma + \frac{1 - F_{c_u}(\tilde{y}_u - \sigma)}{f_{c_u}(\tilde{y}_u - \sigma)} = \tilde{y}_e + c_e, \tag{A.19}$$

where, given A2.(i), the ratio in the left-hand side is defined. The following proposition establishes the property of interest of the equilibrium settlement offer in this specification of the negotiation stage.

PROPOSITION A5. *Consider an initial equilibrium of NEG_2, as well as a change in the distribution of the potential arbitration award that leaves constant $\tilde{y}_u - \tilde{y}_e$. Then, under Assumption A2, such a change does not affect the set of backing-down costs that leads the union to reject the employer's offer.*

Proof. First apply the change of variable $\tau \equiv \tilde{y}_u - \sigma$ to rewrite (A.19) as

$$\tau - \frac{1 - F_{c_u}(\tau)}{f_{c_u}(\tau)} = \tilde{y}_u - \tilde{y}_e - c_e. \tag{A.20}$$

Hold c_e fixed at an arbitrary value. Then, by assumption, the right-hand side of (A.20) is constant. Meanwhile, Assumption A2.(ii) guarantees that the derivative of the left-hand side of (A.20) with respect to τ is strictly positive. Therefore, τ must be constant as well. Recall that $F_{c_u}(\tau) \equiv F_{c_u}(\tilde{y}_u - \sigma)$ is the probability that the union rejects the employer's offer, resulting in arbitration. Thus, given any c_e, the marginal value of c_u—that is, the value which makes the union indifferent between accepting and rejecting the employer's proposal—is fixed.

□

As in Proposition A4, a direct implication of Proposition A5 is that the equilibrium probability that the case reaches the arbitration stage remains constant given a change in the distribution of the arbitration awards that does not affect the difference $\tilde{y}_u - \tilde{y}_e$.

F.2. **Empirical Evidence: The Parties' Signal and the Certainty Equivalent of Arbitration.** We now leverage the structural estimates for the arbitration stage model presented in the main text to assess the relationship between the parties' signal, s_p, and the certainty equivalents of going to arbitration for the union and the employer. With this intent, for each dispute in ARB_F, we compute the parties' certainty equivalents conditional on s_p, based on 10,000 simulated observations.[4]

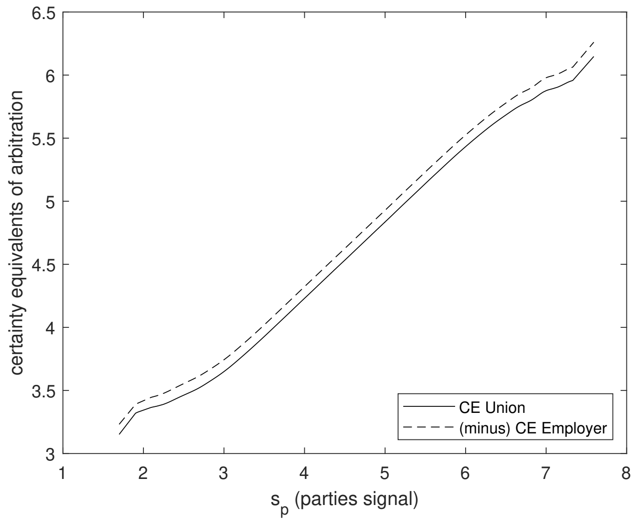
Figure A6 illustrates the estimated conditional certainty equivalents for one particular dispute in our sample, concerning the wage increase of Atlantic City police officers in the year 1993. As shown in the figure, the relationship between the parties' signal and the union's certainty equivalent is positive—that is, in expectation, the

[4]To compute the conditional certainty equivalents, we apply the following procedure separately for each of the disputes in ARB_F: first, we draw 10,000 simulated combinations of s, the ideal wage increase; and s_p. For each such combination, we compute the parties' final offers and the probability that the union wins arbitration, which suffice for us to obtain the expected payoffs for the union and the employer. We then transform the expected payoffs to obtain the parties' certainty equivalents conditional on s and s_p. Finally, we run kernel regressions of the certainty equivalents on s_p to compute the certainty equivalents conditional on s_p only; in the kernel regressions, we employ the Gaussian kernel and bandwidths given by Silverman's rule of thumb.

union gets better off by receiving a higher value of s_p. Conversely, the employer expects to pay more as s_p increases. What is notable about the figure is that the union's gains almost exactly compensate the employer's losses, so the gap between the union's certainty equivalent and the negative of the employer's certainty equivalent remains essentially constant as s_p varies.

We now verify whether the pattern concerning the difference in the conditional certainty equivalents between the union and the employer appears more generally throughout our sample. For each dispute in ARB_F, we compute the gap between the parties' certainty equivalents pointwise over a grid of 10,000 values of s_p, and then take the numerical derivative of this gap with respect to s_p. Considering the distribution of the resulting numerical derivatives over all observations in ARB_F and all values of s_p, the 1st, 2.5th, 10th, 90th, 97.5th, and 99th percentiles are -0.05, -0.01, 0.00, 0.00, 0.01 and 0.03, respectively. For reference, the median values of the conditional certainty equivalent levels for the union and the employer across all observations and values of s_p are 7.29 and 7.49, respectively. As these numbers make clear, the derivatives of the certainty-equivalence gap are heavily concentrated in the neighborhood of zero—implying that, in the disputes in our sample, the signal received by the parties does not affect the gap between their certainty equivalents of arbitration in an important manner. Thus, in light of the theoretical results from Section F.1, the empirical findings reported here suggest that the sample of disputes in ARB_F does not suffer from substantial selection on signals of cases into arbitration.

Figure A6. Certainty equivalents of arbitration conditional on parties' signal



Notes: Figure displays kernel regression estimates of the certainty equivalents of arbitration for the union and the employer on the signal received by the parties, s_p. In the regressions, we employ the gaussian kernel, and the bandwidth is selected by Silverman's rule of thumb. Each regression uses 10,000 simulated observations of the certainty equivalent and s_p, which we compute based on the estimated model primitives for a dispute occurring in 1993 in Atlantic City, NJ. See text and Footnote 4 for details on the simulation.

Appendix G. Ideal Wage Increase and External Efficiency Measures

Throughout the paper, we assume that the arbitrator's objective is to select an award as close as possible to the "ideal" wage increase, defined as the wage increase that would maximize the "interests and welfare of the public" according to the New Jersey statutes. In our analysis in Section 6.3, we interpret the distance between the ideal wage increase and the award actually chosen by the arbitrator as a measure of efficiency. Under such an interpretation, we are able to simulate the model and compute the expected distance between the chosen and the ideal awards under different counterfactual scenarios—which, in particular, allows us to compare the relative efficiency of the conventional and final-offer arbitration formats.

We never directly observe the ideal wage increase for any individual dispute in our data. Instead, we recover the *distribution* of ideal wage increases across cases through our estimation procedure. We believe that it is an important feature of our analysis that we are able to keep an agnostic stance on what constitutes the ideal wage increase for each case. Indeed, a core component of our model is that all players—the arbitrator, the employer, and the union—are uncertain about what the ideal wage increase is in their specific dispute, so it seems proper to treat the welfare-maximizing award as being also hidden to the analyst.

That said, in the present note, we show evidence that some external measures that are often used to assess the efficiency of government policies at the municipal level respond to the outcomes of arbitration in a way that is consistent with our notion of an ideal wage increase. Our chief intention with this exercise is to provide validation for our modeling assumptions beyond that in Section 2 of the paper. We begin by examining results by Mas (2006) on the performance of New Jersey police departments following arbitration. We then present new results on the relationship between arbitration outcomes and local real estate prices.

G.1. **Arbitration and Police Performance.** Mas (2006) investigates how police performance responds to the outcomes of final-offer arbitration in New Jersey, focusing on the same time period that we consider in our analysis in the main text.[5] He documents a drop in the per capita number of crimes cleared by the police in the months following arbitration in disputes that result in a win for the employer. This reduction in the clearance rate corresponds to over 15 cases per 100,000 capita, or approximately 12 percent of the post-arbitration clearance rate experienced by municipalities in which the union wins. The effect seems to persist for at least 22 months after the end of the dispute. Among other measures of police performance, the paper also finds that decisions against the police union in final-offer arbitration are associated with a 5.5 percent increase in reported crime at the municipal level.

Even more pertinent to our analysis, Mas goes on to examine the reason behind this drop in police performance following a union loss in arbitration. He specifically considers the hypothesis that union losses would tend to generate compensation schemes for the police that are below a perceived "reference" or "fair" wage, leading to low satisfaction among the police officers and to their poor performance at work. To investigate this possibility, he proposes using the midpoint between the final offers by the union and the employer as a proxy for the ideal arbitration award in each dispute. Mas then investigates how police performance changes as the realized arbitration award differs from such a proxy. Specifically, using his sample of municipalities that undergo final-offer arbitration, he nonparametrically regresses the post-dispute change in the crime clearance rate on the gap between the arbitration award and the offer midpoint. Figure V of his paper plots the estimated relationship between these

[5] Both our ARB_F data and the sample in Mas (2006) are based on the original data set made available by Ashenfelter and Dahl (2012). But Mas focuses on negotiations between municipalities and police officers, whereas we also include negotiations with fire officers. Also, since our paper and his address distinct research questions, differences arise in the specific set of variables used in the two analyses—which, due to the dropping of observations containing missing values, leads to small differences in the final estimation samples.

two variables.[6] The figure indicates that, for disputes in which the union loses arbitration, the change in the clearance rate tends to substantially increase as the award set by the arbitrator approaches the midpoint between the offers made by the two parties. Once the award crosses the mark established by the offer midpoint, however, the relationship flattens—that is, further increases in the award have no impact on the expected change in clearance rates. These patterns provide support to the notion that there exists some "reference" or "fair" wage and that wage increases up to the approximate location of that point have an impact on police morale and performance. Setting the wage increase below the reference thus poses a cost to society in the form of reduced policing and increased crime. Conversely, by setting a wage increase beyond the reference, society would see no additional police performance gains, and would only bear the fiscal cost of the increased police wage bill. Such a notion of a reference arbitration award corroborates the statutory notion of an ideal award as one that promotes the interest and welfare of the public, which is the concept of an ideal wage increase adopted in our theoretical model.

G.2. **Arbitration and Real Estate Prices.** The potential implications for society of arbitration decisions concerning the compensation of police and fire officers go far beyond the effects of awards on police performance and crime. Among other dimensions that these decisions are likely to affect figure the local government's fiscal health, as well as the morale and performance of other public employees in the same municipality. In this Section, we seek to evaluate the broader consequences of arbitration outcomes in our setting by assessing the relationship between arbitration awards and a measure of real estate values. Such a relationship is especially interesting given our purposes, as housing prices tend to reflect a large set of local amenities and public goods (Rosen, 1979). We also discuss evidence that our estimated structural model

[6] See Mas (2006), p. 810.

captures additional information about the ideal wage increase beyond that offered by a simple offer midpoint.

Taking a cue from Mas (2006), we perform an exercise of similar style, but substituting the municipality's taxable property value for the measures of police performance. The taxable property value, which we obtain with an annual frequency from the New Jersey Data Book, is the estimated true value of all taxable property at the municipality level, based on the evaluation of municipal tax assessors and on the actual sale prices of transacted properties.[7]

We calculate the percentage change in the taxable property value per capita in the three years following the disputes in our ARB_F data, relative to the year that precedes the dispute. For dispute i, denote such a change by $\Delta_{val,i}$. These taxable property values are in real terms, expressed in 1983 dollars. In our analysis below, we focus on arbitration cases that occur in isolation within a four-year window. Specifically, we exclude cases that follow another arbitrated dispute in the same municipality in the previous year; similarly, we exclude cases that precede another arbitrated dispute in the same municipality within the following three years. Given these criteria, our sample consists of 119 cases, in total.[8]

For each case i, we define the *award gap*, denoted by y_{gap}, as

$$y_{gap,i} \equiv y_i - \frac{y_{u,i} + y_{e,i}}{2},$$

[7]More details on the computation of this variable are available on the New Jersey Data Book website: https://njdatabook.rutgers.edu/about-the-data. We include the taxable property value as a covariate in the estimation of the structural model in the main text; more precisely, we incorporate to our specifications there the log of the taxable property value per capita, which we refer to in the text as "log of taxable property per capita."

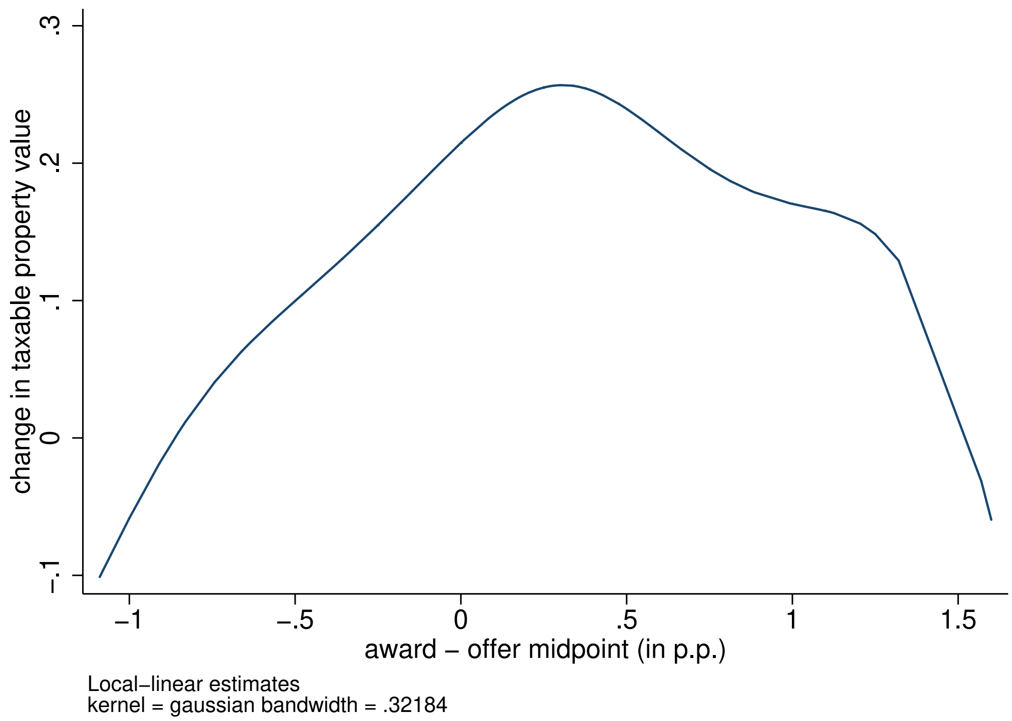
[8]The ARB_F data includes disputes decided up to 1995, and, as explained in footnote 20 in the main text, the taxable property value is available from the New Jersey Data Book from 1983 onwards. Given the four-year window that we consider, we restrict our sample to disputes that take place from 1984 to 1992. Additionally, we discard seven observations in which the gap between the award and the modified offer midpoint, which we define below, is greater than 1.5 percentage points in absolute value. Finally, we discard three observations that show a change in the taxable property value in the three years following the dispute greater than 200 percent or smaller than minus 50 percent.

where y_i is the award set by the arbitrator; and $y_{u,i}$ and $y_{e,i}$ are the offers made by the union and the employer, respectively. We then run a kernel regression of $\Delta_{val,i}$ on $y_{gap,i}$, where we employ the gaussian kernel and calculate the bandwidth using cross-validation. Figure A7 displays the estimated relationship between both variables. The conditional expectation of the taxable property value increases in $y_{gap,i}$ up to a point a little above zero, and decreases from that point onwards. The pattern resembles the findings by Mas concerning the relationship between the crime clearance rate and the award-offers gap. But here the conditional expectation shows more symmetry—which is perhaps not surprising, since the variation in taxable property values captures both some of the negative effects of insufficiently high wage increases (e.g., unmotivated police and fire officers, higher crime rates) as well as the implications of overly high wage increases (e.g., the financial burden on taxpayers).

As noted above, in Figure A7, the conditional mean of the taxable property value achieves its maximum at a strictly positive value of y_{gap}, suggesting that, in expectation, the arbitration award coinciding with the highest increases in property values is greater than the offer midpoint. In fact, our structural model estimates from the main text indicate that, taking an average across the disputes in our sample, the ideal wage increase is 0.16 points greater than the offer midpoint in expectation. Though suggestive rather than conclusive, this evidence is in line with our estimated model capturing additional information about the ideal wage increase beyond that offered by a simple offer midpoint.

In light of the results above, we propose adjusting the offer midpoint to achieve a better proxy for the ideal arbitration award. Specifically, for each case i, define the *modified offer midpoint*, denoted by $\bar{y}_i^{mod}$, as $\bar{y}_i^{mod} \equiv (y_{u,i} + y_{e,i})/2 + 0.16$. Similarly, we define the *modified award gap*, denoted by $y_{gap,i}^{mod}$, as the difference between the award set by the arbitrator and the modified offer midpoint—that is, $y_{gap,i}^{mod} \equiv y_i - \bar{y}_i^{mod}$.

FIGURE A7. Expected property value change conditional on gap between award and offer midpoint



Notes: Figure displays kernel regression estimates of the expectation of the percentage change in taxable property value ($\Delta_{val,i}$) conditional on the gap between the award set by the arbitrator and the offer midpoint. The regression is based on a sample of 119 disputes, as explained in the text. The change in the taxable property value refers to the three years following the dispute, relative to the year that precedes it. The gap between the award and the offer midpoint is expressed in percentage points. We employ the gaussian kernel; the bandwidth is selected by cross-validation.

Table A6 shows the relationship between the modified award gap and the post-arbitration change in taxable property values in a linear regression format. Specifically, we estimate the following specification by OLS:

$$\Delta_{val,i} = \gamma_1 + \gamma_2 y^{mod}_{gap,i} \times \mathbb{1}\{y^{mod}_{gap,i} \leq 0\} + \gamma_3 y^{mod}_{gap,i} \times \mathbb{1}\{y^{mod}_{gap,i} > 0\} + \xi_i,$$

where $\mathbb{1}\{\cdot\}$ denotes the indicator function. The estimated γ_2 and γ_3 in column (1) of the table reaffirm the patterns from Figure A7. That is, the taxable property value

TABLE A6. Property value change and gap between award and expected ideal wage

	(1)	(2)
$y^{mod}_{gap,i} \times \mathbb{1}\{y^{mod}_{gap,i} \leq 0\}$	0.309	0.236
	(0.095)	(0.135)
$y^{mod}_{gap,i} \times \mathbb{1}\{y^{mod}_{gap,i} > 0\}$	-0.172	-0.207
	(0.099)	(0.108)
Union wins		0.079
		(0.117)
Observations	119	119
R^2	0.062	0.065
Adjusted R^2	0.046	0.041

Notes: This table reports OLS results. The unit of observation is a case. In all specifications, the dependent variable is $\Delta_{val,i}$, the percentage change in taxable property value in the three years following the case, relative to the year prior to the dispute. See the text for details on the computation of the modified award gap, $y^{mod}_{gap,i}$. Standard errors are provided in parentheses. The regression is based on a sample of 119 disputes, as explained in the text.

increases in $y^{mod}_{gap,i}$ up to the point $y^{mod}_{gap,i} = 0$, and decreases past that point. The coefficients γ_2 and γ_3 are significant at the one and ten percent levels, respectively. Column (2) reports the result of a similar specification, except that, following Mas, we include a dummy indicating whether the arbitrator chooses the union's final offer as the award. The results in column (2) are very similar to those in column (1).

Of course, one should be careful interpreting the results in this Section, as they are not based on exogenous variation in the arbitrator's award or the parties' offers. We cannot, therefore, take these findings at face value as the causal effects of arbitration outcomes on the evolution of real estate prices. Nonetheless, it is intriguing that the municipalities with wage increases most similar to the expected ideal wage increase, according to our estimates, are also the ones experiencing the best real estate performance on average. Thus, the overall patterns shown in Figure A7 and Table A6 are consistent with the assumption in our model that there exists a unique ideal wage increase—and that departures from it in any direction can be costly to society.

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